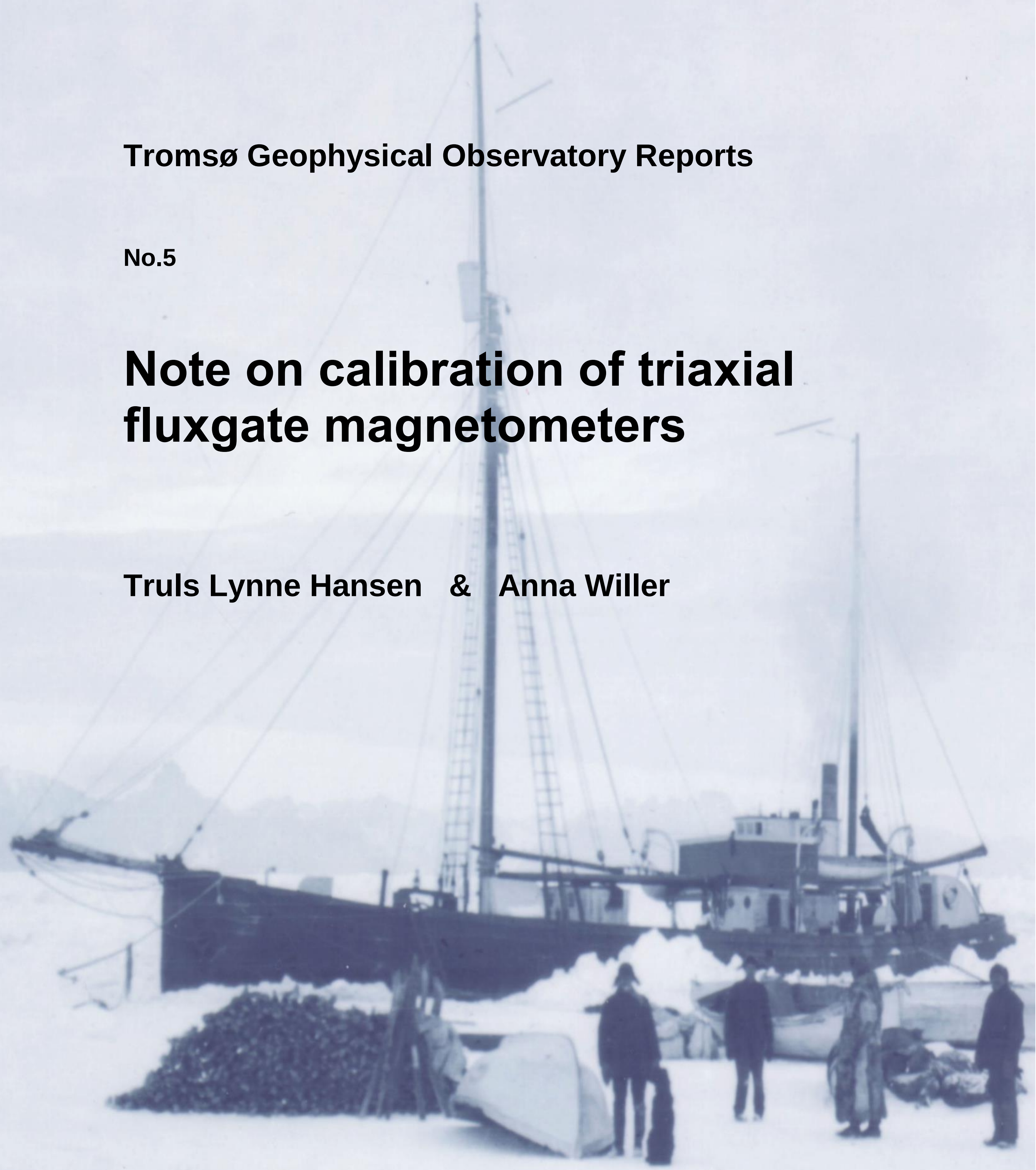
Tromsø Geophysical Observatory Reports

No.5

Note on calibration of triaxial fluxgate magnetometers

Truls Lynne Hansen & Anna Willer



UiT – The Arctic University of Norway 2020

Cover photo: "Conrad Holmboe", Greenland, 1923. In 1922, the precursor of Tromsø Geophysical Observatory, The Geophysical Institute, acquired a 96' steamer to service field stations in the arctic. The boat became trapped in the ice on the eastern seaboard of Greenland in 1923 while unsuccessfully attempting to relieve the personnel there, and, irrevocably damaged by the ice, she limped to Iceland and was scrapped.

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&

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UIT – The Arctic University of Norway, 2020

Foreword

There are somewhere between 150 and 200 magnetic observatories in the world. In short, the aim of these observatories is to monitor Earth´s magnetic field both at short and long time scales locally, and to put the obtained data into a wider and global context through sharing with the larger community. Geomagnetic data is applied in many different fields, such as basic and applied research within Space Physics, Space Weather, Space Climate, Solid Earth physics, Zoology, and so on. The use of the data covers topics from development of large, global models to case studies of phenomena of very localized nature, via monitoring and forecasting to protect modern society from space weather hazards. Since global coverage of magnetic observatories is desirable and crucial for these applications, the operation of magnetic observatories may be viewed as a solidary and common task shared among the world´s countries, and, thus, every nation has a moral duty to measure and monitor the magnetic field within its territory.

Considering the rather sparse amount of magnetic observatories, and the fact that many institutions and countries have several each, there are even fewer people involved in the seldom discussed and often underestimated task of day to day treatment and calibration of geomagnetic data. This has led to a very limited availability of written material covering the topic. In many, perhaps most, cases the knowledge and procedure of transforming the raw voltages coming out of a magnetometer into a meaningful and absolute representation of Earth´s magnetic field at the observatory, has been inherited from one person to the next. This is certainly the case at Tromsø Geophysical Observatory.

Truls Lynne Hansen developed through his career an in-depth knowledge and insight into the calibration of magnetometers. He led the transition from use of the classic variometers with optical registration to fluxgate magnetometers at the Auroral Observatory, and was, thus, among the pioneers who had to understand and create a new way of thinking following the use of new technology. When Truls retired in 2013, I was left in charge of the Norwegian magnetometer network. From him I received his software tools, several hand-written sheets of paper describing the mathematics behind the calibration of fluxgate magnetometers as performed in Tromsø¸ and a promise of writing everything up in a report. The tools and sheets he gave me have been crucial for me in order to secure and continue the provision of quality magnetometer data from Norway, and I am very grateful for the knowledge Truls was able to transfer to me.

Around the same time as Truls retired, Anna Willer was in a similar situation as myself, taking over the management of the magnetometers operated by DTU Space in Greenland and Denmark. Together Truls and Anna have done what Truls promised, and in an excellent manner here presents a thorough treatment of the mathematics needed to make sense of magnetometer data.

Truls and Anna are, furthermore, contributing to and strengthening the long-standing relationship between the geomagnetic communities in Norway and Denmark, through their friendship and common effort to complete this important work. I am infinitely grateful to them both.

Tromsø, April 8th 2020
Magnar Gullikstad Johnsen

Note on calibration of triaxial fluxgate magnetometers

1. Introduction

The triaxial fluxgate magnetometer has over the last decades become the standard instrument for continuous recording of Earth's magnetic field vector at ground. This simple and reliable device now constitutes the core of most geomagnetic observatories and has made possible today´s numerous networks of magnetic stations for ionosphere and magnetosphere studies.

The output of a triaxial magnetometer is three voltage values, one from each sensor, which must be converted to magnetic field values by some mathematical procedure. This procedure requires a set of parameters specifying sensor scale values and offsets as well as orientation in space. This note describes the conversion procedure applied to the magnetometer networks of Tromsø Geophysical Observatory (TGO) at University of Tromsø, Norway, and DTU Space at Technical University of Denmark; for details see https://tgo.uit.no and https://dtu.space.dk

The triaxial fluxgate magnetometer plays the role as variometer at various types of stations: geomagnetic observatories, simple field installations for ionosphere studies or something in between. The requirements depend on type of installation, but the basic procedure for calibration and conversion of the sensor output to magnetic field components remains the same. So, although the emphasis in this disposition is on magnetic observatories, the principles are applicable to less sophisticated setups.

Our concern is not the mechanical and electronic designs of fluxgate magnetometers, only the basic fact they constitute an assembly of three orthogonal sensors, each being sensitive to the field component along the sensor axis. The magnetometer electronics with signal detection circuits and subsequent A/D-conversion are physically separated from the sensor elements, but they are in our context logically included in term ´sensor´.

2. The single sensor

We first consider a single sensor in Earth´s magnetic field and introduce the following notation:

F – the strength of Earth's magnetic field

V – the vertical component of the field

H – the horizontal component of the field

D – the magnetic declination (azimuth of Earth´s field)

I – the inclination of the field

$\zeta_S = \mathrm{I} + \frac{\pi}{2}$ – the zenith distance of the field direction

B_S – the component of Earth´s field vector along the sensor S

ζ_S – the zenith distance of the sensor direction

φ_S – the azimuth of the sensor direction

Δ_S – the angle between the sensor and Earth´s field

The field experienced by the sensor is

$$\mathrm{B_s} = \mathrm{F} * \cos \Delta_S \tag{2-1}$$

Calculating $\cos \Delta_S$ by spherical trigonometry we get

$$\mathrm{B_S} = \mathrm{F} * (-\cos \zeta_\mathrm{S} * \sin \mathrm{I} + \sin \zeta_\mathrm{S} * \cos \mathrm{I} * \cos(\mathrm{D} - \varphi_\mathrm{S})) \tag{2-2}$$

$\mathrm{B_S}$ generates an output signal $\mathrm{U_S}$, and we assume a linear relation between them:

$$\mathrm{B_S} = k_S * U_S + S_0 \tag{2-3}$$

The constant k_S denotes the scale value of the sensor and the S_0 an offset of the zero level. This offset is normally subject to manual adjustment in order to keep $\mathrm{U_S}$ within the measuring range of the AD-converter. It resembles the *baseline* in the classical photographically recording magnetometers. We will later adopt that designation in a modified sense. How to determine the baseline values is the main issue of this note.

3. The triaxial sensor assembly

We denote the three axes X, Y and Z[1] and apply (2-2) and (2-3) to each of them:

$$\mathrm{B_X} = \mathrm{F} * (-\cos \zeta_\mathrm{X} * \sin \mathrm{I} + \sin \zeta_\mathrm{X} * \cos \mathrm{I} * \cos(\mathrm{D} - \varphi_\mathrm{X})) \tag{3-1}$$

$$\mathrm{B_Y} = \mathrm{F} * (-\cos \zeta_\mathrm{Y} * \sin \mathrm{I} + \sin \zeta_\mathrm{Y} * \cos \mathrm{I} * \cos(\mathrm{D} - \varphi_\mathrm{Y})) \tag{3-2}$$

$$\mathrm{B_Z} = \mathrm{F} * (-\cos \zeta_\mathrm{Z} * \sin \mathrm{I} + \sin \zeta_\mathrm{Z} * \cos \mathrm{I} * \cos(\mathrm{D} - \varphi_\mathrm{Z})) \tag{3-3}$$

$$\mathrm{B_X} = \mathrm{k_X} * \mathrm{U_X} + \mathrm{X_0} \tag{3-4}$$

$$\mathrm{B_Y} = \mathrm{k_Y} * \mathrm{U_Y} + \mathrm{Y_0} \tag{3-5}$$

$$\mathrm{B_Z} = \mathrm{k_Z} * \mathrm{U_Z} + \mathrm{Z_0} \tag{3-6}$$

[1] Not to be confused with the cartesian geographic co-ordinates X Y Z often used in geomagnetism. Our XYZ are merely names of the sensors.

We observe that four parameters are needed to describe the output of each sensor, two for the direction (azimuth and zenith distance) and two for the electronic part (scale factor and offset). Thus, in principle, 12 values are required to calculate the outputs of a tri-axes sensor assembly. The three axes assumed being orthogonal, i.e. angle between any two sensors being 90 degrees, the number of directional parameters is reduced from six to three and the total to six.

The values of the scale factors are calculated from the specifications of the electronics components involved or measured in a laboratory setup.

As Earth´s magnetic field is given in a topocentric geodetic reference frame, the orientation of the sensor assembly must be tied to that frame too. That implies using direction of gravity and true north when aligning the sensor assembly. The direction of gravity is readily at hand, but geographic north involves an astronomical or geodetical determination of the azimuth of a sensor. Luckily, as demonstrated in the two following sections, this non-trivial operation can be circumvented by indirectly using the magnetic field as directional reference and restrict the orientation of the sensor assembly to a narrow range of positions

Finally, we carry out a complete absolute measurement of the field with a DI-theodolite and proton precession magnetometer and compute the three offsets by the mathematical model defined by equations 3-1 through 3-6. In our parlance this operation is *the absolute calibration* or simply *the calibration*.

The calibration assures the output of the sensor assembly matches the absolute observation and – hopefully – also produces correct values generally. In case the sensor assembly is influenced by phenomena not accounted for by the model, the offsets inevitably are adjusted so that we still have a match between absolute observation and variometer output.

4. The DHV-mount

This is the traditional setup of the sensor assembly, probably used at an overwhelming majority of world´s magnetic stations, most of the TGO/DTU stations included. The setup procedure runs as follows:

- the offset current in the Y-sensor is disconnected.
- the plane spanned by the X and Y-sensors is made horizontal. The Z-sensor then becomes vertical. The Z-offset is adjusted so that U_Z is near zero.
- the sensor assembly is rotated around the vertical axis until the output U_Y is zero. The X-sensor now points to magnetic north with an azimuth D_0 and the Y-sensor points to magnetic east. The X-offset is adjusted until U_X is near zero.

With this setup the Y-sensor closely tracks variations in D, the X-sensor follows H and the Z-sensor gives us V, hence the designation *DHV- mount*.

The orientation of the sensors are now

$\zeta_X = \frac{\pi}{2} \qquad \varphi_X = D_0$

$\zeta_Y = \frac{\pi}{2} \qquad \varphi_Y = D_0 + \frac{\pi}{2}$

$\zeta_Z = \pi$ $\qquad \varphi_Z$ is undefined. Z is positive downwards.

and the equations in section 3 take the form:

$$B_X = F * \cos I * \cos(D - D_0) = H * \cos(D - D_0) \tag{4-1}$$

$$B_Y = F * \cos I * \sin(D - D_0) = H * \sin(D - D_0) \tag{4-2}$$

$$B_Z = F * \sin I = V \tag{4-3}$$

$$B_X = k_X * U_X + X_0 \tag{4-4}$$

$$B_Y = k_Y * U_Y + Y_0 \tag{4-5}$$

$$B_Z = k_Z * U_Z + Z_0 \tag{4-6}$$

In the relations above there are four unknown quantities, X_0, Y_0, Z_0, and D_0, but only three equations, the system thus being underdetermined. It is common practice to solve this problem by defining Y_0 to be zero because the Y-sensor´s offset current is disconnected.

In the following we will, for the sake of clarity, use superscript ^{abs} to mark quantities that are output of an absolute measurement, i.e. the measured components of the field and the corresponding outputs of the sensors.

With $Y_0 = 0$ the equations (4-1) through (4-6) the output of a calibration is:

$$\boldsymbol{D_0} = \mathbf{D^{abs}} - \mathbf{arcsin}\left(\frac{\mathbf{k_Y * U_Y^{abs}}}{\mathbf{H^{abs}}}\right) \approx \mathbf{D^{abs}} - \frac{\mathbf{k_Y * U_Y^{abs}}}{\mathbf{H^{abs}}} \tag{4-7}$$

$$\boldsymbol{X_0} = \boldsymbol{H^{abs}} * \mathbf{cos}\left(\boldsymbol{D^{abs}} - \boldsymbol{D_0}\right) - \mathbf{k_X} * \mathbf{U_X^{abs}} \tag{4-8}$$

$$\mathbf{Z_0} = \mathbf{F^{abs}} * \mathbf{I^{abs}} - \mathbf{k_Z} * \mathbf{U_Z^{abs}} = \mathbf{V^{abs}} - \mathbf{k_Z} * \mathbf{U_Z^{abs}} \tag{4-9}$$

We will use the term *baselines* for the quantities X_0, Z_0 and D_0.

It is the angular baseline D_0 that makes the DHV-mount viable. As the determination of D^{abs} is based on the astronomically determined azimuth of the observatory´s mire, the azimuth of the X-sensor, and thereby $\boldsymbol{D_0}$ is tied to true north. The Z-sensor being aligned with the direction of gravity, we know the orientation of the sensor assembly in the geodetic frame of reference, and have thus got around the need for an independent observation of the true azimuth of a sensor.

However, we know that Y_0 is not zero. Even with the offset current shut off, a small offset intrinsic to the sensor of the order of 10 nT may remain in the type of sensors we are using [2].

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[2] According to Lars W. Pedersen at DTU Space and Bjørn Ove Husøy at TGO, the engineers building our magnetometers. We are using sensors from Pandect with antiparallel rod cores.

This is not negligibly; for instance,[3] with H=17200 nT it corresponds to a shift of 2´ in D. Additionally, as pointed out in sections 7 and 8, pillar differences and sensor misalignments inevitably also contribute to the sensor offsets, presumably more than the intrinsic offset.

A nonzero Y_0 will shift D_0 away from magnetic north. The reading of $\mathrm{U_Y}$. will not be zero when the Y-sensor is pointing magnetic east, and we therefore have to rotate the assembly a small angle Y_0/H further so that the Y-sensor experiences a small contribution from H which cancels Y_0 and brings U_Y to zero.

If we can´t simply skip Y_0, we are left with an underdetermined system of equations. Fortunately, there is a loophole available. The equations for the Y-sensor (4-2 and 4-5) give us

$$\mathrm{k_Y} * U_Y + Y_0 = \mathrm{H} * \sin(D - D_0) \tag{4-10}$$

In geomagnetic observations the angle $(D - D_0)$ only rarely exceeds a few degrees., so we make a third order expansion of the sine function:

$$\mathrm{k_Y} * \mathrm{U_Y} + \mathrm{Y_0} \approx \mathrm{H} * \left(\mathrm{D} - \mathrm{D_0} - \frac{(\mathrm{D}-\mathrm{D_0})^3}{6}\right) \tag{4-11}$$

The third order term will amount to tiny 0.2” if $|\mathrm{D} - \mathrm{D_0}| = 1°$, grows to 5” at 3°and 23” at 5°. This means ignoring the third order term will cause an error of less than half an arcminute in even the most extreme excursions in declination. That is certainly good enough in any scientific application of D-recordings.

As for absolute calibration we note that the accuracy of an absolute measurement of D is around 10” and that hardly any calibration is carried out when $|\mathrm{D} - \mathrm{D_0}|$ is larger than 3°. Thus, the third order term causes not problem, and we can use the following expression for D-calibration:

$$\mathrm{D_0} + \frac{\mathrm{Y_0}}{\mathrm{H^{abs}}} = \mathrm{D^{abs}} - \frac{\mathrm{k_Y} * \mathrm{U_Y^{abs}}}{\mathrm{H^{abs}}} \tag{4-12}$$

Neither of the two terms on the left-hand side can be determined separately. We are able to find the sum only and D_0 and Y_0/H coalesce into one computable quantity. Thereby the problem of underdetermination vanishes for all variations in declination relevant to geomagnetism. However, as demonstrated later, this that not mean that the value of Y_0 is irrelevant.

Equation (4-12) implies that an absolute calibration does not give us D_0 but a modified version which we will denote *effective* D_0:

$$\mathrm{D_0^*} = \mathrm{D_0} + \frac{\mathrm{Y_0}}{\mathrm{H^{abs}}} \tag{4-13}$$

and the calibrating equation for declination becomes

[3] The magnetic field component values in our numerical examples are typical for the Brorfelde geomagnetic observatory: D = 3.5°, H = 17200 nT, V = 47200 nT, I = 70.0°, F = 50200 nT

$$\mathbf{D_0^* = D^{abs} - \frac{k_Y * U_Y^{abs}}{H^{abs}}} \quad (4\text{-}14)$$

Physically the X-sensor azimuth is D_0, but mathematically we have to use the virtual azimuth D_0^* to compensate for the shift of D_0 during the setup procedure; D_0^* is the azimuth the X-sensor would have got if Y_0 was zero.

The baseline X_0 is also affected by the non-zero Y_0. Replacing D_0 in equation (4-1) with D_0^* we obtain the effective baseline for the X-axis

$$X_0^* = H^{abs} * cos\left(D^{abs} - D_0^*\right) - k_X * U_X^{abs} \quad (4\text{-}15)$$

The Z-sensor is not affected by $Y_0 \neq 0$. Summed up, the calibration equations for the non-zero Y_0 case are

$$\boldsymbol{D_0^*} = \mathbf{D^{abs} - \frac{k_Y * U_Y^{abs}}{H^{abs}}} \quad (4\text{-}16)$$

$$\mathbf{X_0^* = H^{abs} * cos\left(D^{abs} - D_0^*\right) - k_X * U_X^{abs}} \quad \mathbf{(}4\text{-}17)$$

$$\boldsymbol{Z_0^* = Z_0} = \mathbf{F^{abs} * I^{abs} - k_Z * U_Z^{abs} = V^{abs} - k_Z * U_Z^{abs}} \quad (4\text{-}18)$$

From a mathematical point of view little has changed here compared to the $Y_0 = 0$ case. In practice we can indeed forget about the effective baselines and use equations (4-7), (4-8), and (4-9) without any concern over Y_0. The effective baselines merely serve as a reminder of why the arcsine has vanished from equation (4-7), and that Y_0 after all is not zero.

We have only vague ideas of the magnitude of Y_0 There are contributions from the intrinsic sensor offset, pillar difference (section 8), and sensor misalignment (section 9). Daring to make an educated guess, we would say somewhere between 20 and 200 nT, in a well-designed geomagnetic observatory closer to the lower limit rather than the upper.

Tacitly, we have from the very beginning assumed Y_0/H^{abs} be to a small term, in fact it is required. Equation (4-12) is valid under the condition that

$$|D - D_0| < \Lambda \quad (4\text{-}19)$$

where Λ is at most a few degrees. Introducing D_0^* we have

$$\left|D - D_0^* + \frac{Y_0}{H^{abs}}\right| < \Lambda \quad (4\text{-}20)$$

and taking the positive values of $(D - D_0)$, we get

$$D - D_0^* < \Lambda - \frac{Y_0}{H^{abs}} \quad (4\text{-}21)$$

Now, $D - D_0^* = \frac{k_Y * U_Y}{H}$ being the variation in declination we are recording, this means a nonzero Y_0 is restricting the range available for linear recording. In a worst case Y_0 may in fact reduce the positive side of the range to nil. A similar effect is seen for negative $(D - D_0)$.

Clearly the value of Y_0/H^{abs} must be a fraction only of Λ to allow for reasonable variations in the declination. Taking Λ=3°, a value like 0.5° might be a reasonable upper limit for Y_0/H^{abs}. That implies Y_0 should be less than 150 nT when H=17200 nT. So, Y_0 probably does not represent a problem at well-designed observatories outside the polar region. At high latitude observatories however distortions in the output may occur; at the Qaanaaq observatory with H=4000 nT, the critical value of Y_0 will be as low as 35 nT.

The value of D_0 was fixed in the setup of the sensor assembly, but D are subject to a secular variation. The value of $(D - D_0)$ can thus become unacceptably large over time, making an adjustment of D_0 necessary. The value of Y_0/H^{abs} will keep almost constant, only slightly adjusted each time a new calibration is carried out.

Now, finally, with the baselines at hand, the output of the DHV-mount can be computed

$$\mathbf{D} = \boldsymbol{D_0^*} + \frac{\mathbf{k_Y} * \boldsymbol{U_Y}}{\boldsymbol{H}} \tag{4-22}$$

$$\mathbf{H} = \frac{\boldsymbol{X_0^*} + \boldsymbol{k_X} * \boldsymbol{U_X}}{\mathbf{cos}(\mathbf{D} - \boldsymbol{D_0^*})} \tag{4-23}$$

$$\mathbf{V} = \mathbf{B_Z} = \mathbf{k_Z} * \mathbf{U_Z} + \boldsymbol{Z_0^*} \tag{4-24}$$

Be aware that, in principle, the formula

$$\mathrm{H} = \sqrt{\mathrm{B_X}^2 + \mathrm{B_Y}^2} \tag{4-25}$$

and the corresponding one for F are forbidden unless $\mathrm{Y_0}$ is known.

5. An alternative: DIF-mount

We start out with a DHV-mount where not only the offset current in the Y-sensor, but also in the X-sensor is disabled. The sensor assembly is rotated around the Y-sensor axis until $\mathrm{B_X}$ is close to zero. The Z-sensor is then almost parallel to Earth´s magnetic field, the output representing the total field. The X-sensor, now nearly perpendicular to the field, will be a measure of the inclination. The Y-sensor provides the declination as for the DHV-mount. This DIF arrangement is used at several TGO stations, including the Tromsø observatory.

This means we have

$$\zeta_X = I_0 \qquad \varphi_X = D_0$$

$$\zeta_Y = \frac{\pi}{2} \qquad \varphi_Y = D_0 + \frac{\pi}{2}$$

$$\zeta_Z = I_0 + \frac{\pi}{2} \qquad \varphi_Z = D_0$$

The angle I_0 has replaced X_0 as baseline and will take values close to the field´s inclination at the site. Gravity is still used to make the Y-sensor horizontal, but we have renounced aligning the Z-sensor with direction of gravity, and are instead indirectly via the field inclination, using the plumb line in the DI-theodolite as vertical reference. The price paid for this is leaving $\mathrm{X_0}$ undetermined. The advantage is a sensor mount with a close to one to one correspondence between the sensor output and the magnetic element observed with the DI-flux and proton magnetometer. With a continuously running a proton magnetometer we have a good running check of the Z-sensor, and we can if needed make use of absolute measurements of D or I alone. In particular at field operations under difficult conditions this might be useful.

The sensor output now take the form

$$\mathrm{B_X} = -\mathrm{F} * \sin \mathrm{I} * \cos \mathrm{I_0} + \mathrm{F} * \cos \mathrm{I} * \sin \mathrm{I_0} * \cos(\mathrm{D} - \mathrm{D_0}) \quad (5\text{-}1)$$

$$\mathrm{B_Y} = \mathrm{F} * \cos \mathrm{I} * \sin(\mathrm{D} - \mathrm{D_0}) \quad (5\text{-}2)$$

$$\mathrm{B_Z} = \mathrm{F} * \sin\mathrm{I_0} * \sin\mathrm{I} + \mathrm{F} * \cos\mathrm{I_0} * \cos \mathrm{I} * \cos(\mathrm{D} - \mathrm{D_0}) \quad (5\text{-}3)$$

$$\mathrm{B_X} = \mathrm{k_X} * \mathrm{U_X} + X_0 \quad (5\text{-}4)$$

$$\mathrm{B_Y} = \mathrm{k_Y} * \mathrm{U_Y} + Y_0 \quad (5\text{-}5)$$

$$\mathrm{B_Z} = \mathrm{k_Z} * \mathrm{U_Z} + \mathrm{Z_0} \quad (5\text{-}6)$$

In order for this to be a determined system of equation we need to get rid of X_0 as well as Y_0.

The calibration of the Y-sensor runs is exactly like the DHV case:

$$\mathbf{D_0^*} \approx \mathbf{D^{abs}} - \frac{\mathbf{k_Y * U_Y^{abs}}}{\mathbf{H^{abs}}} \quad (5\text{-}8)$$

D_0^* replaces D_0 in equations (5-1), (5-2), and (5-3) in order to account for the effect of Y_0 not being zero.

$\mathrm{I_0}$ is found from equation (5-1). With a second order approximation of $\cos(\mathrm{D} - D_0^*)$ we have

$$\frac{\mathrm{B_X^{abs}}}{\mathrm{F^{abs}}} \approx \sin\left(\mathrm{I_0} - \mathrm{I^{abs}}\right) - \frac{1}{2}\left(\frac{\mathrm{D^{abs}} - D_0^*}{2}\right)^2 * \sin(\mathrm{I_0} + \mathrm{I^{abs}}) \quad (5\text{-}8)$$

$\mathrm{I^{abs}}$ being close to $\mathrm{I_0}$ we expand the sine:

$$\sin\left(\mathrm{I_0} - \mathrm{I^{abs}}\right) \approx \left(\mathrm{I_0} - \mathrm{I^{abs}}\right) - \frac{\left(\mathrm{I_0} - \mathrm{I^{abs}}\right)^3}{6} \quad (4\text{-}9)$$

Accepting an error of 1.5” we can skip the last term if $\left(\mathrm{I_0} - \mathrm{I^{abs}}\right) < 2°$. That restriction will suffice for most practical cases. Then taking $\mathrm{I_0} + \mathrm{I^{abs}} \approx 2 * \mathrm{I^{abs}}$ the calibrating equation becomes

$$\mathbf{I_0^*} = \mathbf{I^{abs}} + \frac{\mathbf{k_X * U_X^{abs}}}{\mathbf{F^{abs}}} + \frac{\mathbf{1}}{\mathbf{8}}\left(\mathbf{D^{abs}} - \boldsymbol{D_0^*}\right)^{\mathbf{2}} * \mathbf{sin}\left(\mathbf{2} * \mathbf{I^{abs}}\right) \quad (5\text{-}10)$$

where the effective baseline I_0^* encompasses a contribution from the unspecified X_0 .

$$\mathrm{I_0^*} = \mathrm{I_0} + \frac{\mathrm{X_0}}{\mathrm{F^{abs}}} \quad (5\text{-}11)$$

The last term in (5-10) will be small, but far from negligible, with $\mathrm{D^{abs}} - \mathrm{D_0} = 3°$ and $\mathrm{I} = 70°$ it amounts to 45”.

The value of $\mathrm{X_0}$ presumably is of the same size of order as $\mathrm{Y_0}$ and restricting the linear range for inclination. However, the effect is of less concern that for declination since $\mathrm{F^{abs}}$ never takes low values.

With I_0^* replacing I_0, Z_0^* is derived from (5-3)

$$\boldsymbol{Z_0^*} = \mathbf{F^{abs}} * \mathbf{sinI_0^*} * \mathbf{sinI^{abs}} + \mathbf{F^{abs}} * \mathbf{cosI_0^*} * \mathbf{cos\,I^{abs}} * \mathbf{sin}\left(\mathbf{D^{abs}} - \mathbf{D_0^*}\right) - \mathbf{k_Z} * \mathbf{U_Z^{abs}} \quad (5\text{-}12)$$

Now the output from the varometer can be computed as follows

$$\mathbf{D} = \mathbf{D_0^*} + \frac{\mathbf{k_Y * U_Y}}{\mathbf{H}} \quad (5\text{-}13)$$

$$\mathbf{I} = \mathbf{I_0^*} - \frac{\mathbf{k_x * U_X}}{\mathbf{F}} - \frac{\mathbf{1}}{\mathbf{8}}(\mathbf{D} - \mathbf{D_0^*})^{\mathbf{2}} * \mathbf{sin\,2I_0^*} \quad (5\text{-}14)$$

$$\mathbf{F} = \frac{\mathbf{k_Z * U_Z} + \boldsymbol{Z_0^*}}{\mathbf{sin}\boldsymbol{I_0^*} * \mathbf{sinI} + \mathbf{cos}\boldsymbol{I_0^*} * \mathbf{cos\,I} * \mathbf{sin}\left(\mathbf{D} - \boldsymbol{D_0^*}\right)} \quad (5\text{-}15)$$

Strictly, the equations for D, I and F are coupled and should be solved simultaneously. However, a good estimate of H will solve the problem.

Not knowing Y_0 and X_0 the formula

$$\mathbf{F} = \sqrt{\mathbf{B_X}^{\mathbf{2}} + \mathbf{B_Y}^{\mathbf{2}} + \mathbf{B_Z}^{\mathbf{2}}} \quad (5\text{-}16)$$

and the corresponding one for H are in principle forbidden.

6. The effect of scale value inaccuracy

We use the horizontal component from DHV-variometer to illustrate the effects of an inaccurate scale value. Other components and DIF mount give similar results. To this end a simplified version of (4-22) is sufficient:

$$\mathrm{H} = \mathrm{k_X} * \mathrm{U_x} + \mathrm{X_0} \quad (6\text{-}1)$$

Here $\mathrm{X_0}$is the result of an absolute observation

$$\mathrm{X_0} = \mathrm{H^{abs}} - \mathrm{k_X} * \mathrm{U_X^{abs}} \quad (6\text{-}2)$$

so that

$$\mathrm{H} = \mathrm{k_X} * \left(\mathrm{U_X} - \mathrm{U_X^{abs}}\right) + \mathrm{H^{abs}} \quad (6\text{-}3)$$

A small error $\delta\mathrm{k_x}$ in $\mathrm{k_X}$ and $\delta\mathrm{H^{abs}}$ in $\mathrm{H^{abs}}$ give rise to an error in H

$$\delta\mathrm{H} = \delta\mathrm{k_X} * \left(\mathrm{U_X} - \mathrm{U_X^{abs}}\right) - \delta\mathrm{H^{abs}} \quad (6\text{-}4)$$

Introducing the relative error of the scale value we get the convenient form

$$\delta\mathrm{H} = \frac{\delta\mathrm{k_X}}{\mathrm{k_X}} * \left(\mathrm{H} - \mathrm{H^{abs}}\right) - \delta\mathrm{H^{abs}} \quad (6\text{-}5)$$

This expression first reflects the trivial, albeit important, fact that the value of $\delta\mathrm{k_X}/\mathrm{k_X}$ has little effect on readings of H which are in the vicinity of $\mathrm{H^{abs}}$, δH^{abs} then being the important term.

Let´s take δH^{abs} around 0.5 nT and $\delta k_X/k_x$ equal to 1% as typical values. With these numbers, a reading of H with $(H - H_{abs})$ = 50 nT have an error of 1 nT, equally contributed by the two sources. Higher values of $(H - H^{abs})$ will soon make the scale factor error dominant. Presumably most values of H^{abs} are within a hundred nT from the quiet value of the field, at least outside the polar region. Thus improving $\delta k_X/k_X$ somewhat beyond 1% seems worthwhile.

When computing mean values comprising a many absolute observations – like annual means – the average $(H - H_{abs})$ is expected to be quite small since negative and positive value at least partially will cancel out. Large field excursion of several hundred nT are so rare that they do not affect the error in the mean value. To this end the accuracy of the scale value is not important, the crucial term is the uncertainly in the absolute observations.

In ionosphere applications we often are deal with large excursions in the field. However, even with $(H - H^{abs})$ = 500 nT δH will amount to only 5 nT. In an ionosphere physics context that is no serious problem, and $\delta k_X/k_X$ around 1% is satisfactory.

7 Effect of pillar differences

In sections 3 to 5 we implicitly assume the calibration measurements are done at the same place as the variometer. That is never possible; the sensor assembly is always sitting at a pillar at some distance from the theodolite pillar, and thus is not seeing the same field. Nevertheless, not knowing the difference we have no choice but to hope for the best and proceed using the formulae in section 4 or 5 for calculations of the baselines.

A pillar difference shifts the output of the sensors. Let Δ_S be the component of the vector difference between the two pillars along the sensor S. Equation (2-3) then takes the modified form

$$B_S = k_S * U_S + S_0 + \Delta_S \qquad (7\text{-}1)$$

where B_S now is the output of the sensor be it at the absolute pillar. When calibrating we find the sum $S_0 + \Delta_S$ only and therefore keep the notation S_0 for the sum as well; we say that Δ_S is incorporated in S_0. The alternative, mathematically correct procedure, would be introducing a specific name for S_0 without pillar difference and yet another for S_0 including pillar difference. As we will encounter similar ´incorporations´ several times, that will only lead to a plethora of unimportant variables. The S_0 emerging from the calibration process incorporates all effects equivalent to a field along the sensor which are not a part of the calibrating field. In this perspective equation (7-1) merely informs us that Δ_S is part of S_0.

For a DHV-mount the incorporation of Δ_S is of no concern for the X- and Z-sensor. For the Y-sensor, however, Δ_Y will contribute to Y_0 with the potential danger of making it too large. For a DIF-mount there is a similar problem for X_0 as well. Magnetic station in areas with large local anomalies could be affected by this.

8. Effects of sensor misalignments.

The sensors are rarely pointing exactly as presumed in our model: there may be small deviations from orthogonality, and the assembly may not be perfectly oriented. Below we will make first order approximations to the effect of such deviations. We use the following notation to describe them:

μ_S – a small deviation in zenith distance from the correct angle for sensor S

ν_S- a small deviation in azimuth from the correct angle for sensor S

The magnetic field values in our numerical examples are taken from the Brorfelde observatory, see footnote 3.

We first apply this to the Y-sensor in a DHV-mount. Equation (3-2) then takes the form

$$\mathrm{B_Y} = -\mathrm{V} * \cos\left(\frac{\pi}{2} + \mu_\mathrm{Y}\right) + \mathrm{H} * \sin\left(\frac{\pi}{2} + \mu_\mathrm{Y}\right) * \cos\left(\mathrm{D} - \mathrm{D_0} - \frac{\pi}{2} - \nu_\mathrm{Y}\right) \tag{8-1}$$

which in a first order approximation reduces to

$$B_Y \approx H * (D - D_0 - \nu_\mathrm{Y}) + V * \mu_\mathrm{Y} \tag{8-2}$$

The angle ν_Y is per definition zero as azimuth of the Y-axis is found by an absolute measurement of declination. There is no such convenient way to get rid of μ_Y. The term V$*\ \mu_\mathrm{Y}$ tells us the obvious fact that the output of the Y-sensor is contaminated by the vertical component when $\mu_\mathrm{Y} \neq 0$.

As $\mathrm{B_Y} = \mathrm{k_Y} * \mathrm{U_Y} + Y_0$ we have

$$\mathrm{Y_0} - \mu_Y * \mathrm{V} = \mathrm{H} * (\mathrm{D} - \mathrm{D_0}) - \mathrm{k}_Y * \mathrm{U_Y} \tag{8-3}$$

In a calibration the term $\mu_Y * \mathrm{V}$ thus behaves like part of Y_0. Estimating μ_Y to be $0.1°$ we get $V * \mu_\mathrm{Y} = 82\ nT$ when V=47200 nT. Recalling the discussion following equation (4-21), 82 nT alone does not cause any non-linear output, but when combined with other contributions to $\mathrm{Y_0}$ it may do so. Efforts to bring μ_Y below 0.1° when adjusting to horizontality at the setup therefore is advisable.

In a calibration to find the declination baseline equation (8-2) is rewritten to

$$\mathrm{D_0^*} = \mathrm{D_0} + \frac{\mathrm{Y_0}}{\mathrm{H^{abs}}} - \mu_\mathrm{Y} * \mathrm{tgI^{abs}} = \mathrm{D^{abs}} - \frac{\mathrm{k_Y} * \mathrm{U_Y^{abs}}}{\mathrm{H^{abs}}} \tag{8-4}$$

which means $\mu_\mathrm{Y} * \mathrm{tg}I^{abs}$ is incorporated in D_0^*.

Provided the absolute measurements are carried out at reasonably quiet magnetic conditions, the average of the contamination field is concealed in baseline D_0^* and do not affect the recording of D. However, small variations in $\mu_\mathrm{Y} * \mathrm{tg}I^{abs}$ from one absolute observation to the next will manifest as a noise in the series of baselines; with $\mu_\mathrm{Y} = 0.1°$ and I=70° a disturbance of 10´ in the inclination will be seen as a 11” shift in D_0^*

The same exercise on the equation for the X-sensor in a DHV-mount yields

$$\mathrm{B_X} = -\mathrm{V} * \cos\left(\frac{\pi}{2} + \mu_\mathrm{X}\right) + \mathrm{H} * \sin\left(\frac{\pi}{2} + \mu_\mathrm{X}\right) * \cos(\mathrm{D} - \mathrm{D_0} + \nu_\mathrm{X}) \tag{8-5}$$

The last term vanishes in a first order approximation giving us

$$B_X \approx H + \mu_X * V \qquad (8\text{-}6)$$

We observe the trivial fact that the output from the X-sensor is contaminated by the vertical component when not perpendicular to it. Applying V=47200 nT and μ_X =0.1° we get $\mu_X * V$ = 82 nT, as for the Y-sensor.

The equation for calibration (in the first order version) is now

$$X_0^* - \mu_X * V^{abs} = H^{abs} - k_X * U_X^{abs} \qquad (8\text{-}7)$$

which demonstrates that $\mu_X * V^{abs}$is incorporated into X_0^* without complications.

As for the Y-sensor V^{abs} will not the same in all calibrations giving rise to small variations in the baseline X_0^*. However, even a very large excursion like 500 nT will cause a shift of merely 1 nT.

Finally, the Z-sensor:

$$B_Z = -V * \cos(\pi + \mu_Z) + H * \sin(\pi + \mu_Z) * \cos(D - \varphi_Z + \nu_Z) \qquad (8\text{-}8)$$

The factor $\boldsymbol{\cos(D - \varphi_Z + \nu_Z)}$ can assume any value from -1 to 1 depending on how the sensor is tilted, effectively it is unpredictable. Take the worst case of 1 first order version of (8-8) becomes

$$B_Z = V - H * \mu_Z \qquad (8\text{-}9)$$

As anticipated B_Z is affected by H if the sensor is not precisely vertical. With H=17200 nT and $\mu_Z = 0.1°$ the effect is 39 nT.

The approximated calibration equation is

$$Z_0 + H^{abs} * \mu_Z = V^{abs} - k_Z * U_Z^{abs} \qquad (8\text{-}10)$$

illustrating that $H^{abs} * \mu_Z$ becomes part of the baseline, and, similar to for the two other axes, that here will be slight shifts in the baseline due to variations in H^{abs}.

In a DIF-mount the Y-sensor the behaves exactly as in the DHV-mount.

For the X-axis in a DIF-mount equation (3-1) takes the form

$$\frac{B_X}{F} = -\sin I * \cos(I_0 + \mu_X) + \cos I * \sin(I_0 + \mu_X) * \cos(D - D_0 - \nu_X) \qquad (8\text{-}11)$$

Because I_0 is determined through an absolute observation, μ_X is zero. The azimuth error then vanishing in a first order approximation, there are no appreciable contamination or baseline “noise” in the output of this sensor.

At last, the Z-sensor in the DIF-mount: equation (5-3) with the directional errors is

$$B_Z = F * \sin(I_0 + \mu_Z) * \sin I + F * \cos(I_0 + \mu_Z) * \cos I * \cos(D - D_0 - \nu_Z) \qquad (8\text{-}12)$$

In usual first order approximation it turns into

$$\mathrm{B_Z} \approx \mathrm{F} * \cos(I - I_0) + F * \mu_Z * (I - I_0) \qquad \text{(8-13)}$$

which means that the sensor is seeing a contamination from the inclination. It is normally quite small: with F=50200, $\mu_Z = 0.1°$, and $(I - I_0) = 10'$ it amounts to only 0.25 nT.

The corresponding calibration equation is

$$\mathrm{Z_0} - \mathrm{F^{abs}} * \mu_\mathrm{Z} * \left(\mathrm{I^{abs}} - \mathrm{I_0}\right) = \mathrm{F^{abs}} * \cos\left(\mathrm{I^{abs}} - \mathrm{I_0}\right) - \mathrm{k_Z} * \mathrm{U_Z^{abs}} \qquad \text{(8-14)}$$

where the tiny quantity $\mathrm{F^{abs}} * \mu_\mathrm{Z} * (I - I_0)$ is incorporated in $\mathrm{Z_0}$. Variations in the inclination from one calibration to another will hardly cause any baseline noise.

Generally, we observe that the gross effects of small misalignments of the order of 0.1° are absorbed by the baselines. Minor shifts in the baselines may however occur due to the field not being the same from one calibration to the next. Special attention should be given to Y_0 in order to avoid nonlinearities.

9. Postscript

As far as we can see the DHV and DIF mounts are the only possible ways to set up a triaxial sensor without an independent geodetic observation of the direction of one of the sensors. Within the limitations described these mounts serve geomagnetic excellently. Nevertheless, while grappling with this text, and $\mathrm{Y_0}$ in particular, we wished we had a sensor assembly with independently observed geodetic orientation, all three axes with identical mathematics to find simple offsets.

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